AREC 815: Experimental and Behavioral Economics

Estimating Distributional Preference Parameters

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Approaches to Parameter Estimation

Maximum Likelihood Estimation

Let y_j be the observed decision in choice situation j for $j = 1, \ldots, J$

$$y_j = g(x; \theta) + \varepsilon_j$$

where x denotes the exogenous parameters of the choice situation (e.g. price), θ denotes the preference parameters, and $\varepsilon_j \sim \mathcal{N}(0, \sigma^s)$

- Subject chooses y_j from a convex choice set
- $g(x; \theta) + \varepsilon_j$ is the **demand function**
  - Derived by solving for utility-maximizing choice

Because $\varepsilon_j \sim \mathcal{N}(0, \sigma^s)$, we know that $\underbrace{y_j - g(x; \theta)}_{\varepsilon_j} \sim \mathcal{N}(0, \sigma^s)$

Maximum Likelihood Estimation

The normal error term characterizes the distribution of y_j:

$$f(y_j|x;\theta) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\left[\left(\frac{y_j - g(x;\theta)}{\sigma}\right)^2/2\right]}$$

$$= \frac{1}{\sigma}\phi\left(\frac{y_j - g(x;\theta)}{\sigma}\right)$$

Knowing $f(y_j|x;\theta)$, we can write down the log-likelihood function for θ:

$$\ell(\theta) = \sum_j \ln\left[f(y_j|x;\theta)\right]$$

$$= \sum_j \ln\left[\frac{1}{\sigma}\phi\left(\frac{y_j - g(x;\theta)}{\sigma}\right)\right]$$

ML Estimation: CES Example

CES other-regarding utility function:

$$u_s(\pi^s, \pi^o) = [\alpha(\pi^s)^\rho + (1-\alpha)(\pi^o)^\rho]^{1/\rho}$$

Interpretation of the model parameters:

- $\hat{\alpha}$: fair-mindedness/selfishness, weight on payoff to *self* vs. *other*
- $\hat{\rho}$: curvature of altruistic indifference curves, measures willingness to trade off equality (payoff difference) and efficiency (sum of payoffs)

Subjects maximize utility s.t. budget constraint $\pi^s + p\pi^o = m$

ML Estimation: CES Example

CES expenditure (e.g. demand) function is given by:

$$s^* = \frac{\pi^s}{m} = \frac{\left(\frac{\alpha}{1-\alpha}\right)^{1/(1-\rho)}}{(p)^{\rho/(\rho-1)} + \left(\frac{\alpha}{1-\alpha}\right)^{1/(1-\rho)}}$$

Subjects choose π^s from convex set; assume normally-distributed errors:

$$s_j = \frac{\pi^s_j}{m_j} = \frac{\left(\frac{\alpha}{1-\alpha}\right)^{1/(1-\rho)}}{(p_j)^{\rho/(\rho-1)} + \left(\frac{\alpha}{1-\alpha}\right)^{1/(1-\rho)}} + \varepsilon_j$$

for $\varepsilon \sim \mathcal{N}(0, \sigma^s)$

ML Estimation: CES Example

To derive the likelihood, we exploit the fact that $\varepsilon_j = s_j - s^*(p; \alpha, \rho, \sigma)$:

$$\ell(\theta) = \sum_j \ln\left[f\left(s_j | p; \alpha, \rho, \sigma\right)\right]$$

$$= \sum_j \ln\left[\frac{1}{\sigma}\phi\left(\frac{s_j - s^*}{\sigma}\right)\right]$$

$$= \sum_j \ln\left[\frac{1}{\sigma}\phi\left(\frac{s_j - \frac{A}{p_j^{\rho/(\rho-1)}+A}}{\sigma}\right)\right]$$

where

$$A = \left(\frac{\alpha}{1-\alpha}\right)^{1/(1-\rho)}$$

ML Estimation: CES Example

This likelihood function is implemented in PS1, Question 7:

```
function [ll]=ll_ces(param)

% Declare GLOBAL variables
global obs share price

alpha=param(1,1);
rho=param(2,1);
sigma=param(3,1);

num=(alpha/(1-alpha))^(1/(1-rho));
num=num.*ones(obs,1);
denom=price.^(rho/(rho-1))+num;
dens=(normpdf((share-num./denom)/sigma))/sigma;
dens=max(dens,0.00000001);
ll=-sum(log(dens),1);
```

ML Estimation: Adjusting for Censoring

What if $s^* > 1$? How do we adjust for censoring ($C_j = 1$)?

$$\ell(\alpha, \rho, \sigma) = \sum_j \ln\left[\left\{(1 - C_j) \cdot f(s_j | p; \alpha, \rho, \sigma) + C_j \cdot \Pr[s_j = 1 | p; \alpha, \rho, \sigma]\right\}\right]$$

$$= \sum_j \ln\left[\left\{(1 - C_j) \cdot \frac{1}{\sigma}\phi\left(\frac{s_j - s^*}{\sigma}\right) + C_j \cdot \Pr\left[s_j^* + \varepsilon_j > 1\right]\right\}\right]$$

$$= \sum_j \ln\left[\left\{(1 - C_j) \cdot \frac{1}{\sigma}\phi\left(\frac{s_j - s^*}{\sigma}\right) + C_j \cdot [1 - \Phi(1 - s^*)]\right\}\right]$$

Because $\Pr\left[s_j^* + \varepsilon_j > 1\right] = 1 - \Pr\left[\varepsilon_j < 1 - s_j^*\right] = 1 - \Phi(1 - s^*)$

ML Estimation: Adjusting for Censoring

Adjusting for censoring requires a minor modification of the ML code:

```
dens=(normpdf((share-num./denom)/sigma))/sigma;

dist=???
like=???
like=max(dens,0.00000001);

ll=-sum(log(dens),1);
```

ML Estimation: Discrete Outcomes

Subjects choose from a menu of allocations: $a_k \in A$ with K elements

- Example: "simple tests" proposed by Charness and Rabin (2002)

Log-likelihood takes the form:

$$\ell(\theta) = \sum_j \sum_k z_{jk} \cdot \ln\left[P_{jk}(x;\theta)\right]$$

where

- z_{jk} is an indicator for choosing option a_k in choice situation j
- $P_{jk}(x_j;\theta)$ is the probability of choosing a_k in choice situation j

ML Estimation: Additive Random Utility Model

In an additive random utility model, realized utility is the sum of the modeled component ("representative utility") and a random component

$$V_j(a_k|x_j;\theta) = U(a_k|x_j;\theta) + \epsilon_j$$

When ϵ_j is EV1-distributed, the choice probabilities are given by:

$$P_{jk}(x_j;\theta) = \frac{e^{U(a_k|x_j;\theta)}}{\sum_{k\in K} e^{U(a_k|x_j;\theta)}}$$

$$= \frac{1}{1 + \sum_{z\neq k\in K} e^{U(a_z|x_j;\theta) - U(a_k|x_j;\theta)}}$$

When $U(a_k|x_j;\theta)$ is a non-linear function of the structural parameters, normalize by the variance of the logit error term (which can be estimated)

ML Estimation: Summary

Many experiments are motivated by theory

- Experiments are controlled, simplified choice environments
- Characterizing the optimal decision is often straightforward
- Characterizing the likelihood function is straightforward

Experimental design should be linked to the estimation strategy

- Continuous vs. discrete choice sets
- What variation is needed to identify model parameters?

Individual Effort and Fairness

Characterizing Fairness

What constitutes a fair allocation?

- Equality-efficiency tradeoffs represent a spectrum of views on fairness, but entirely ignore issues of entitlement, desert, equity
- Theories of reciprocity parameterize fairness/kindness in terms of where one's material payoff falls in the feasible distribution

Konow (*JEL*, 2003) reviews survey evidence that people reject these simplistic ideals in favor equity/attribution/proportionality/desert

- People should be held accountable for their choices, effort
- People should not be accountable for factors beyond their control
- Not clear exactly how ability differences fit in to these theories

Characterizing Fairness Ideals

Cappelen et al (*AER*, 2007) take this idea seriously, and argue that people are likely to be heterogeneous in their conceptions of fairness

- Egalitarianism
- Libertarianism
- Intermediate ideals: equity theory, accountability principle, etc.
  - Fairness $\Rightarrow$ you are accountable for factors under your control
  - Differences in income stemming from (some subset of) effort, individual choices, innate ability, etc. are fair; however, inequality resulting from factors beyond agents' control is not fair
  - What factors are beyond agents' control?

Characterizing Fairness Ideals

Propose a specific utility formulation:

$$U_i\left(y_i|X\right) = \gamma y_i - \frac{\beta_i}{2X}\left(y_i - m_i(X)\right)^2$$

- X = dictator's budget
- $m_i(X)$ = fairness ideal (i.e. "fair" payoff for i)
- β_i = cost of deviating from fairness ideal
- γ = marginal utility of money relative, to logit error term

Implied optimal (interior) allocation to self:

$$y_i^* = \frac{\gamma}{\beta_i} \cdot X + m_i(X)$$

Characterizing Fairness Ideals

Conduct modified dictator games preceded by team production phase

- Agent i assigned return to investment, a_i
- Choose investment level, q_i
- Total income $X(\mathbf{a}, \mathbf{q}) = a_i q_i + a_j q_j$ is divided between i and j
- Both i and j propose an allocation; one is chosen at random

Implied fair allocation to *other* subject:

- Egalitarianism $\Rightarrow m_i(\mathbf{a}, \mathbf{q}) = X(\mathbf{a}, \mathbf{q})/2$
- Libertarianism $\Rightarrow m_i(\mathbf{a}, \mathbf{q}) = a_i q_i$
- "Liberal egalitarianism" $\Rightarrow m_i(\mathbf{a}, \mathbf{q}) = \frac{q_i}{q_i + q_j} \cdot X(\mathbf{a}, \mathbf{q})$

Characterizing Fairness Ideals: Results

TABLE 1—DESCRIPTIVE STATISTICS OF OFFERS MADE TO OPPONENT

	Offer	
	Share	Amount (in NOK)
Mean	0.271	229
Median	0.292	200
Standard deviation	0.219	219
Minimum	0	0
Maximum	0.75	800

Characterizing Fairness Ideals: Results

Of 96 subjects, 47 always propose the same budget share

- 15 subjects always propose an even split
- 25 always propose to take everything

17 of those subjects who propose the same allocation in both decisions also contribute the same budget share in both decision problems

Characterizing Fairness Ideals: Results

Simple reduced form analysis of allocation decisions:

$$ProposedShare_{ik} = \alpha + \delta \left(\frac{q_{ik}}{q_{ik} + q_{jk}} \right) + \psi \left(\frac{a_{ik} q_{ik}}{a_{ik} q_{ik} + a_{jk} q_{jk}} \right) + \epsilon_{ik}$$

Dependent Variable: Proposed Budget Share to Self

Sample:	**All Subjects**			**Variable**
Investment share	0.26**	·	0.157	0.467***
	(0.128)		(0.159)	(0.144)
Contribution share	·	0.215**	0.121	0.084
		(0.109)	(0.138)	(0.14)
Constant	0.55***	0.573***	0.541***	0.33***
	(0.074)	(0.077)	(0.079)	(0.073)
Budget size controls	Yes	Yes	Yes	Yes
R^2	0.034	0.033	0.038	0.193

Note: robust standard errors clustered at the player level. ∗ ∗ ∗ indicates significance at the 99 percent level; ∗∗ indicates significance at the 95 percent level; and ∗ indicates significance at the 90 percent level.

Estimating the Distribution of Fairness Ideals

A structural model of subjects allocation decisions:

- Dictators choose from finite choice set: $50, 100, 150, \ldots$

  $\Rightarrow$ Discrete choice model

- Utility of allocating y_i to self given by

$$U_i\left(y|a,q\right) = \underbrace{\gamma y_i - \frac{\beta_i}{2X(a,q)}\left(y_i - m_i(a,q)\right)^2}_{=V_i(y|a,q)} + \varepsilon_{iy}$$

  where $m_i(a,q)$ is i's fairness ideal and ε is IID EV1

- Error terms imply logit probability structure

Estimating the Distribution of Fairness Ideals

The probability that i chooses to allocate herself y is:

$$\Rightarrow P_{iy} = \left(\frac{e^{V_i(y|a,q)}}{\sum_{z=0,50,\ldots,X(a,q)} e^{V_i(z|a,q)}} \right)$$

If we knew the parameters $\{\beta_i, m_i(a,q)\}$ for a specific individual i, we could write down an explicit formula for i's choice probabilities

- Conversely, if we had a single subject (with a fixed $\{\beta_i, m_i(a,q)\}$), we could estimate the parameters via maximum likelihood (logit)

Use a **mixed logit** framework to estimate distribution of fairness ideals (e.g. libertarian, egalitarian, liberal egalitarian) within subject population

- People are heterogenous, not enough data to estimate individual parameters; need to impose structure on parameter distributions

Estimating the Distribution of Fairness Ideals

Don't observe individual β_i parameters

- Assume $\ln \beta \sim \mathcal{N}(\zeta, \sigma^2)$; ζ and σ are parameters to be estimated

Primary goal is to estimate λ_k, fraction of subject pool with holding fairness ideal k, where $k \in \{\text{egalitarian}, \text{libertarian}, \text{liberal egalitarian}\}$

- Never know an individual's fairness ideal, only dist'n

Write down choice probabilities in terms of parameters that will govern the distribution of preferences: $\zeta, \sigma, \lambda_E, \lambda_{LE}, \lambda_L$

$$P_{iy} = \sum_k \lambda_k \int \left(\frac{e^{V_i(y|a,q,k,\beta,\gamma)}}{\sum_{z=0,50,\ldots,X(a,q)} e^{V_i(z|a,q,k,\beta,\gamma)}} \right) f(\beta|\zeta,\sigma)\, d\beta$$

$\Rightarrow$ Simulate the integral following methods described in Train (2003)

Characterizing Fairness Ideals

TABLE 2—ESTIMATES OF THE STRUCTURAL MODEL

	Specification			
	1	2	3	4
λ^{SE}, share strict egalitarian	0.435		0.674	0.513
	(0.090)		(0.085)	(0.097)
λ^{LE}, share liberal egalitarian	0.381	0.725		0.487
	(0.088)	(0.085)		(0.097)
λ^{L}, share libertarian	0.184	0.275	0.326	
	(0.066)	(0.085)	(0.085)	
γ, marginal utility of money	28.359	16.437	18.189	22.464
	(3.589)	(1.739)	(2.174)	(2.793)
ζ, mean of $\log(\beta)$	5.385	4.171	4.304	4.585
	(0.349)	(0.412)	(0.459)	(0.365)
σ, standard deviation of $\log(\beta)$	3.371	3.155	3.148	2.897
	(0.530)	(0.507)	(0.498)	(0.448)
Log likelihood	−337.584	−367.958	−366.969	−350.736

Characterizing Fairness Ideals

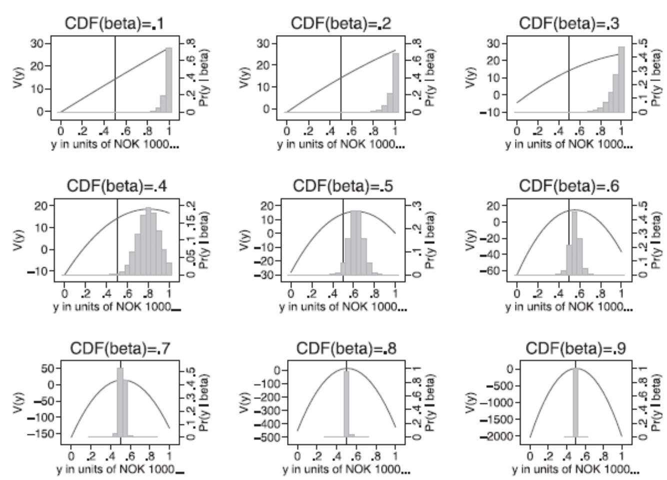


FIGURE 1. IMPLIED CHOICE PROBABILITIES

Notes: Implied choice probabilities are plotted as solid bars for an individual with $m = 0.5$ and deterministic utility, $V(y)$. They are calculated at the deciles of the estimated β distribution using the estimates in the preferred specification 1 in Table 2.

The Development of Fairness Ideals

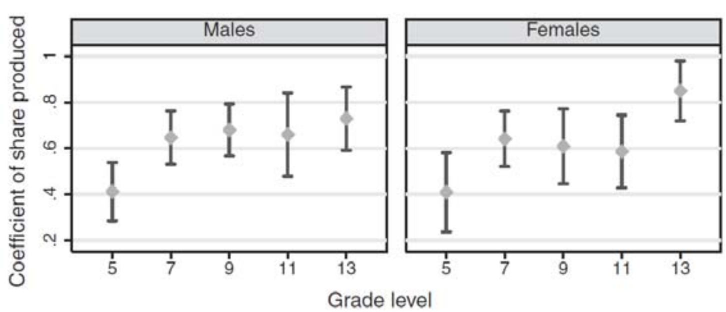
Almas et al (*Science*, 2010) conduct similar experiments with teenagers

- Real effort task, alternative is to play computer games
- Subjects are Norwegian 5^{th} through 13^{th} graders
- Contribution depends on effort and multiplier

Share given and multiplier	Males in grade level (*n*)					Females in grade level (*n*)				
	5th (58)	7th (51)	9th (51)	11th (36)	13th (35)	5th (46)	7th (56)	9th (42)	11th (61)	13th (50)
(A) Share given in first part of experiment										
Share given	0.422 ±0.020	0.449 ±0.017	0.466 ±0.013	0.435 ±0.027	0.448 ±0.028	0.443 ±0.022	0.467 ±0.016	0.457 ±0.014	0.435 ±0.016	0.481 ±0.018

No change in overall generosity as children age

The Development of Fairness Ideals



Older children make allocations (more) contingent on productivity

The Development of Fairness Ideals

	Grade level					
	5th	7th	9th	11th	13th	All
Share of egalitarians	0.636	0.401	0.272	0.267	0.224	0.365
	±0.060	±0.059	±0.057	±0.056	±0.056	±0.027
Share of meritocrats	0.054	0.220	0.363	0.396	0.428	0.287
	±0.037	±0.054	±0.063	±0.069	±0.075	±0.028
Share of libertarians	0.310	0.379	0.364	0.337	0.347	0.348
	±0.057	±0.055	±0.061	±0.059	±0.069	±0.026
Log likelihood	−827.4	−881.4	−797.6	−865.0	−790.3	−4219.7

Egalitarians become more meritocratic as they group up!

Comparing Models of Distributional Preferences

The distributional preference models in Fisman *et al* (2007,2015) and Cappelen *et al* (2007,2010) differ along several key dimensions:

- Price variation vs. variation in relative merit
- Continuous vs. discrete choice sets
- Other differences?